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Design of Multi-loop PI controller for quadruple-tank system with Minimum and Non-Minimum Phase Behaviors

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***Abstract*—Most of the large and complex industrial processes are naturally Multi Input Multi Output (MIMO) systems. MIMO systems in comparison with SISO systems are difficult to control due to inherent nonlinearity and due to the interaction between the system variables. The Quadruple tank process is one of the multivariable system that requires control over at least two variables and is a laboratory process that consists of four interconnected tanks and two pumps.**

In this paper, a multivariable four-tank process has been developed for simulation experiments and it is controlled for both minimum and non-minimum phase cases. Its performance can be affected when system is shifted from minimum to non-minimum phase configuration and vice versa. The mathematical models are constructed from the both physical and simulation data.

The decentralized PI controller has been implemented to control the four-tank process and the performance of the proposed controller is tested for reference tracking and disturbance rejection behavior. All simulation results are simulated using the MATLAB software and comparing different controllers' performance. Simulation results are presented and discussed.

Keywords—Decentralized PI controller, MIMO system, Minimum phase systems, Non minimum phase systems, Four-tank process.

I. Introduction

In process control industry, more than 95% of the control loops are of PI/PID (Proportional-Integral/Proportional-Integral-Derivative) type. Because is still the most commonly used control technique in the process industries and this is mainly attributed to its effectiveness and relatively simple structure, which can be easily understood and implemented in practice. Many industrial controlled problems have more number of manipulated and controlled variables. It is common for industrial processes to have significant uncertainties, strong interaction of minimum and non-minimum phase behavior. Decentralized (or multi-loop) PI control systems are widely used for Multivariable systems control problems in spite of the development of multivariable control strategies [1], [2], [3]. MIMO systems exhibit complex dynamics because of the interactions between input variables and output variables.

The Quadruple Tank System has been used in control literature to illustrate many concepts in multivariable control; particularly performance limitations due to multivariable right half plane zeros. It exhibits elegantly complex dynamics, which emerge from a simple cascade of tanks [4]. To control a quadruple tank system, one essential problem is how to handle the interactions among two loops. An effective approach is to apply the so called decentralized control strategy: each loop is controlled by one controller independently based on local information and local actions.

This paper presents a PI decentralized control methodology for a quadruple tank process with both minimum phase and non-minimum phase system configuration.

The paper is structured as follows: Section 2 describes A benchmark quadruple tank process. The design of controllers for quadruple tank process followed by results and discussions is given in Section 3. Some concluding remarks and future prospects complete the paper.

II. Description process

In this section we derive a mathematical model for the quadruple-tank process from physical data. The quadruple tank experiments consist of four water tanks and two pumps [4] as given in Fig. 1 for a system schematic. The aim is to control the water level in the lower tanks (tank 1 and 2) with the two pumps. The inputs of the process to control are the input voltages of the pumps v_1 and v_2. The outputs are the corresponding water level in the tanks $l_1$and l_2 [5].

There are two valve section in the setup each consisting of two manually operated valves [6]. Changing the flow ratios of the valve sections makes the system minimum or non-minimum phase.

The mathematical model of four tank systems is given as follows,

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$$\frac{dl_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gl_1} + \frac{a_3}{A_1}\sqrt{2gl_3} + \frac{\gamma_1 k_1}{A_1}u_1 \quad (1)$$

$$\frac{dl_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gl_2} + \frac{a_4}{A_2}\sqrt{2gl_4} + \frac{\gamma_2 k_2}{A_2}u_2 \quad (2)$$

$$\frac{dl_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gl_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \quad (3)$$

$$\frac{dl_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gl_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \quad (4)$$

Where,

A_i: is the cross-section area of the tank i;

a_i: is the cross-section area of the outlet hole i;

l_i: is the water level in the tank i;

u_i: is the voltage applied to the pump i;

γ_i: is the valve constant for the valve i;

k_i: is the pump constant for the pump i;

g: the acceleration of gravity;

k_c: the pump gain.

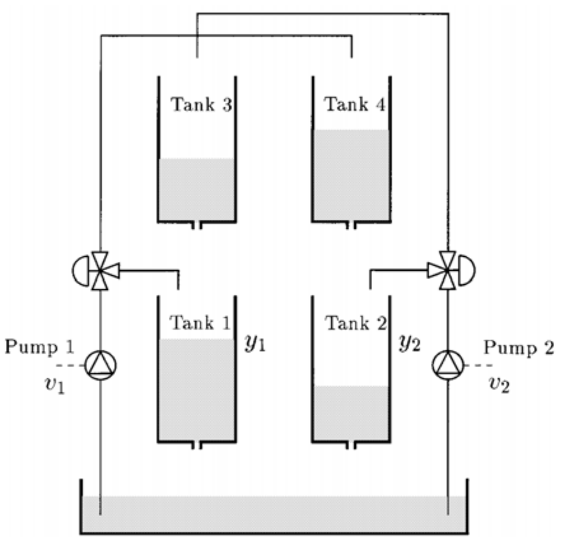

Fig. 1. Schematic of the quadruple tank process [4]

The linearized state space model is [19]:

$$\frac{dl}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_1} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_2} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} l + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u \quad (5)$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} l \quad (6)$$

The corresponding transfer matrix is:

$$G(p) = \begin{bmatrix} \frac{\gamma_1 c_1}{1+pT_1} & \frac{(1-\gamma_2)c_1}{(1+pT_3)(1+pT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+pT_4)(1+pT_2)} & \frac{\gamma_2 c_2}{1+pT_2} \end{bmatrix} \quad (7)$$

The plant can be shifted from minimum to non-minimum phase configuration and vice versa simply by changing a valve controlling the flow ratios γ_1 and γ_2 between lower and upper tanks. The minimum-phase configuration corresponds to $1 < (\gamma_1 + \gamma_2) < 2$ and the non-minimum phase one to $0 < (\gamma_1 + \gamma_2) < 1$.

The main aim of this work is to analyze both minimum and non-minimum phase MIMO systems on the same plant with the presence of decentralized PI controller.

The discrete four-tank plant model is as follows:

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k \quad (8)$$

III. DECENTRALIZED PI CONTROLLER DESIGN

The PI classical controllers are widely used in control engineering practice, and the reason for commonly used control technique is that they are easy to implement. Decentralized or multi-loop PI controllers are used to control MIMO system [7], [8].

In this paper the decentralized PI controller is implemented to control the four-tank process minimum and non-minimum phase process.

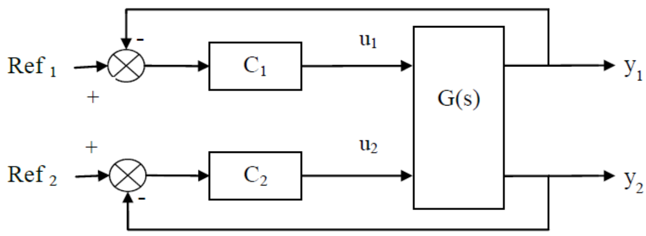

Fig. 2. Decentralized PI Controller

A. Four-tank minimum phase process

The PI controller tuning parameters are chosen in terms to obtain the better performances, $K_{p1} = 4,\ K_{p2} = 3.5,\ T_{i1} = 9\ and\ T_{i2} = 3.5$

The initial levels in tanks 1 and 2 are 9.5 and 10.5 [cm].

The reference is chosen as a square wave.

Simulation results are developed with Matlab codes written to simulate a quadruple tank process.

The process outputs and control input signal, under decentralized PI controller method are shown in Fig. 3.

B. Four-tank non-minimum phase process

The PI controller tuning parameters are chosen in terms to obtain the better performances, $K_{p1} = 1.4,\ K_{p2} = 0.21,\ T_{i1} = 9\ and\ T_{i2} = 10$

The initial levels in tanks 1 and 2 are 12.4 and 13.2 [cm].

The reference is chosen as a square wave.

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Simulation results are developed with Matlab codes written to simulate a quadruple tank process.

The process outputs and control input signal, under decentralized PI controller method are shown in Fig. 4.

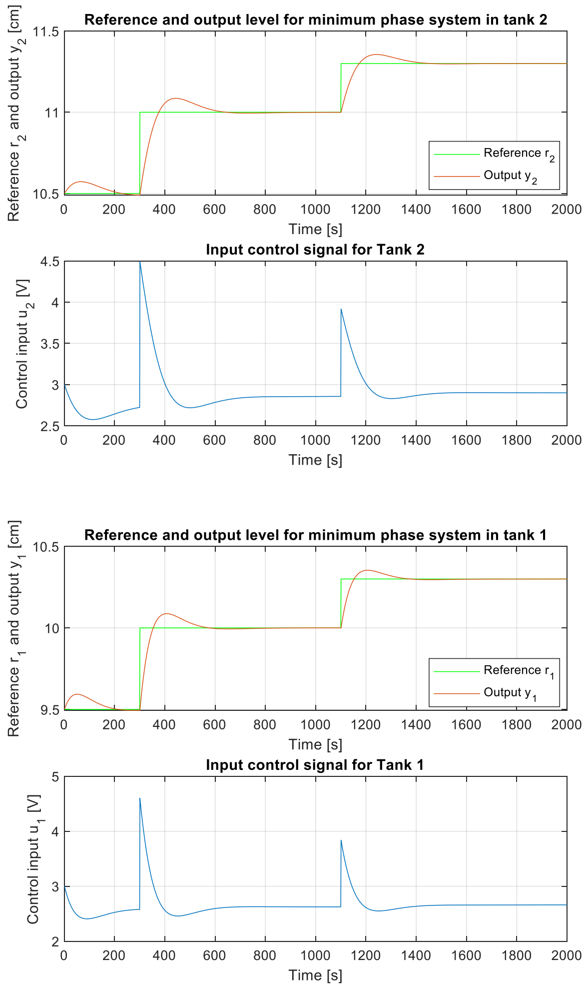

Fig. 3. Output levels for tanks 1 and 2, and control input signal for pumps 1 and 2 for four-tank minimum phase process

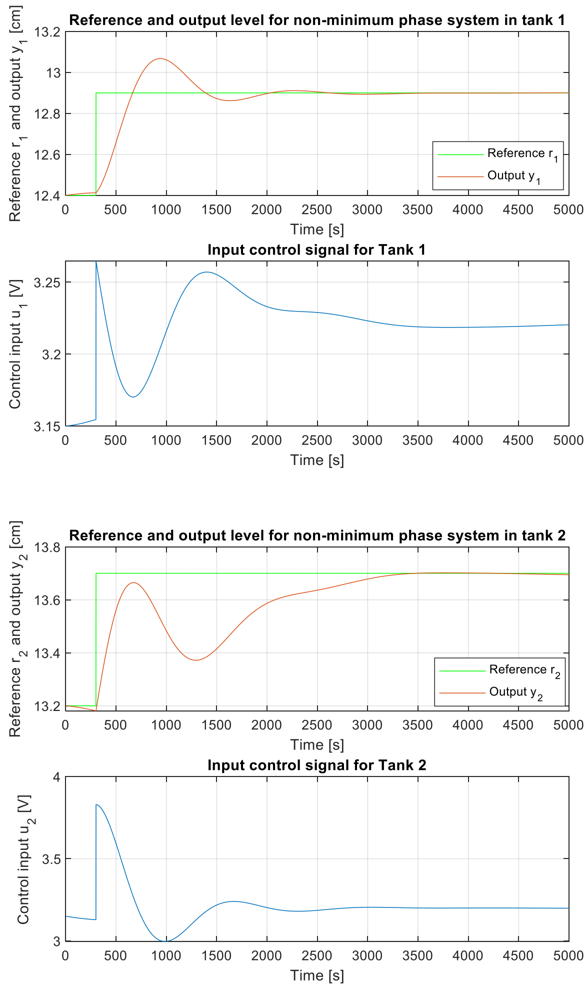

Fig. 4. Output levels for tanks 1 and 2, and control input signal for pumps 1 and 2 for four-tank non-minimum phase process

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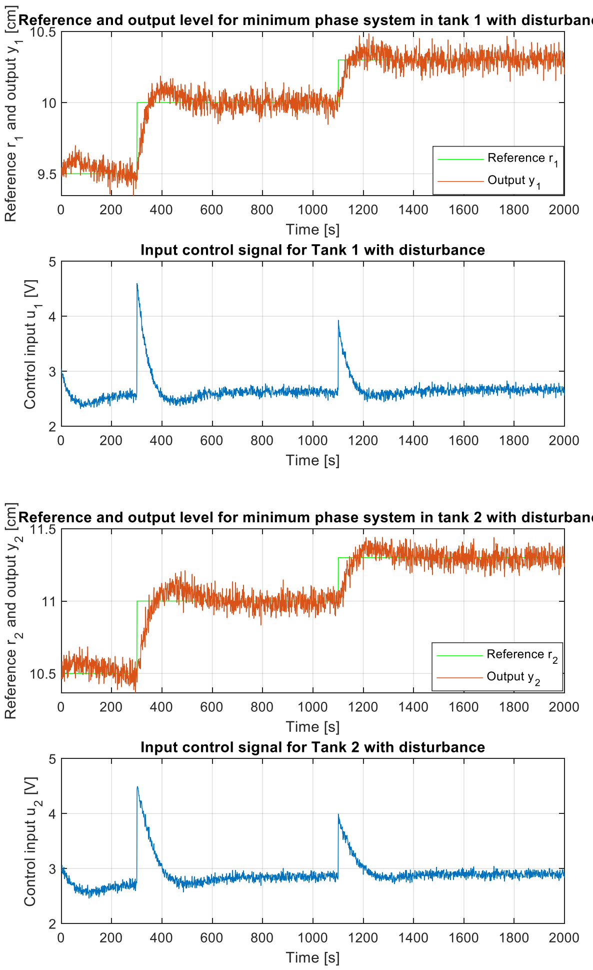

Fig. 5. Output levels for tanks 1 and 2, and control input signal for pumps 1 and 2 for four-tank minimum phase process with a random disturbance

C. Performance analysis

A decentralized PI controller is implemented to achieve the target. A good control performance can be achieved with the proper tuning of the controller constants, but poor performance and controller instability can result from poor choice of values.

The Output for tanks 1 and 2 show that levels are controlled, but the PI controller for four-tank non-minimum phase process (seen Fig.4) takes a long time to follow the reference level, the settling time in this case is ten times larger than the previous case of minimum phase process (seen Fig.3).

It also can be seen from the Output levels for tanks 1 and 2, it is difficult to control the four-tank non-minimum phase process however, the performance of minimum phase process is much better than a non-minimum phase process in this particular case.

When the load disturbance is applied as given in Figures 5 and 5, it can be seen that the proposed method allows guaranteeing the disturbance rejection and the tracking performance is achieved successfully.

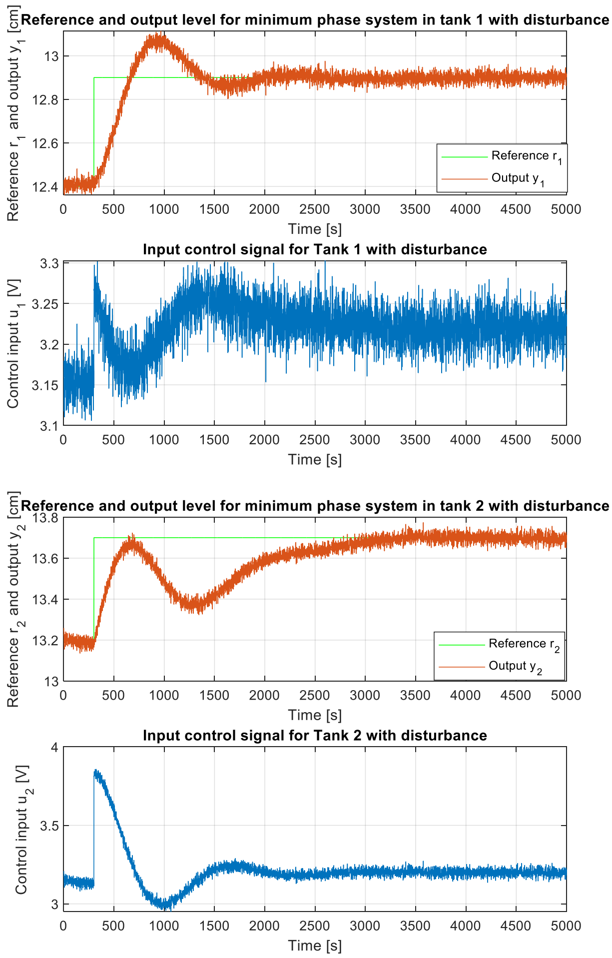

Fig. 6. Output levels for tanks 1 and 2, and control input signal for pumps 1 and 2 for four-tank non-minimum phase process with a random disturbance

IV. CONCLUSION

In this paper, a decentralized PI controller for four-tank process in the minimum and non-minimum phase region, was designed and simulated in MATLAB environment with suitable tuning parameters. From the simulation results, it is clear that, the PI controller, exhibits better performance in the minimum phase process than the non-minimum case in terms of tracking, the height of water in the tank 1 and 2 changed according to the given reference inputs, overshoot and settling time and the effect of disturbance is well rejected in both cases of minimum and non-minimum phase process . The work can be extended to compare with other methods such as the MPC controller version because it is one of the most well studied areas in control systems theory.

REFERENCES

[1] K.J. Åström, *and al.,* The future of PID control Control Engineering Practice, 2001.

[2] J. Lee, *and al.,* Multiloop PI controller tuning for interacting multivariable processes, Computers Chem. Eng. 1998.

[3] Y. Halevi, *and al.,* Automatic tuning of decentralized PID controllers for MIMO processes Process Control, 1997.

[4] K.H. Johanson, The Quadruple-tank process: A multivariable process with an adjustable zero, in IEEE Trans. Contr. Syst. Tech., Vol 8.No. 3, p. 456-465, 2000.

[5] M. Nagarajapandian, S. Kanthalakshmi and T. Anitha, Design and Implementation of Controllers for Quadruple Tank System, Journal of Control & Instrumentation, Vol 9.No. 1, p. 25-32, 2018.

[6] K.H. Johansson, A. Horc and O. Wijk, Teaching Multivariable Control Using the Quadruple-Tank Process, in IEEE conference on Decision and Control, Phoenix, 1999.

[7] A. O'Dwyer, PI and PID controller tuning rules: an overview and personal perspective, Proceeding of the IET Irish Signals and Systems Conference, Dublin Institute of Technology, p. 161-166, 2006.

[8] D. Chen, D.E. Seborg, Design of decentralized PI control systems systems based on Nyquist stability analysis, Journal of Process Control, vol. NO. 13, p. 27-39, 2003.